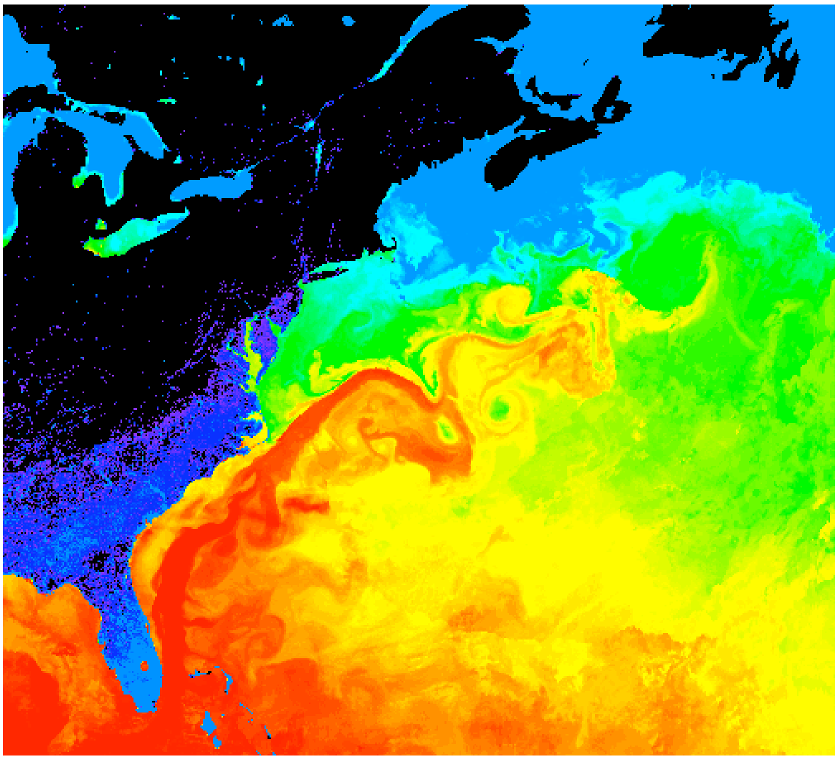
2. HOMOGENEOUS MODEL OF THE WIND-DRIVEN CIRCULATION

The circulation in the different ocean basins contains many common elements, including:

- subtropical and subpolar gyres,
- western boundary currents,
- inertial recirculation,
- separated meandering jets.

This suggests a common dynamical cause, independent of basin geometry.



Snapshot of Sea Surface Temperatures over western North Atlantic

THE HOMOGENEOUS MODEL

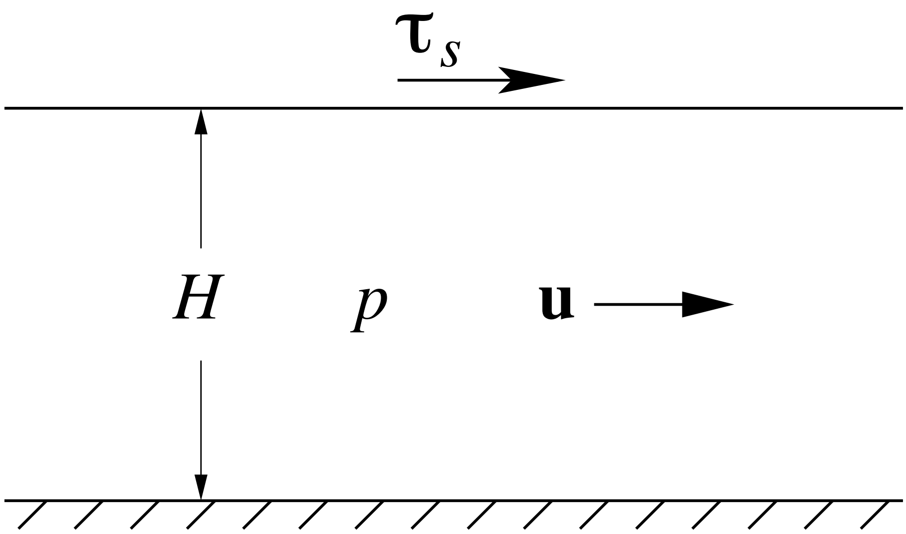
The “classical” model of the wind-driven circulation, and one of the great successes of GFD.

While highly idealised, many underlying ideas carry over to more complete descriptions of the ocean circulation.

Assume:

- uniform density,
- circulation independent of depth,
- ocean of uniform depth,
- dissipation through linear friction,
- β-plane, i.e., $f = f_0 + \beta y$.

The final four assumptions are stronger than strictly necessary, but allow us to considerably simplify the mathematics.



Equations of motion:

$$\frac{\partial u}{\partial t} + \mathbf{u}.\nabla u - fv + \frac{1}{\rho_0}\frac{\partial p}{\partial x} = \frac{\tau_s^{(x)}}{\rho_0 H} - ru, \tag{2.1}$$

$$\frac{\partial v}{\partial t} + \mathbf{u}.\nabla v + fu + \frac{1}{\rho_0}\frac{\partial p}{\partial y} = \frac{\tau_s^{(y)}}{\rho_0 H} - rv, \tag{2.2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.3}$$

where τ_s is the surface wind stress and r is the coefficient of linear friction.

Three equations in three unknowns: u, v and p.

We can eliminate p by forming a vorticity equation, $\partial(2.2)/\partial x - \partial(2.1)/\partial y$, to give:

$$\left(\frac{\partial}{\partial t} + \mathbf{u}.\nabla\right) q = \frac{1}{\rho_0 H}\left\{\frac{\partial \tau_s^{(y)}}{\partial x} - \frac{\partial \tau_s^{(x)}}{\partial y}\right\} - r\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right). \tag{2.4}$$

Here

$$q = f(y) + \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \tag{2.5}$$

is the *absolute vorticity.*

Equation (2.4) contains the three essential ingredients of any ocean gyre:

- a vorticity source (wind stress curl),
- a vorticity redistribution (advection),
- a vorticity sink (friction).

Finally we can use (2.3) to define a streamfunction, ψ, such that

$$u = -\frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \psi}{\partial x}. \tag{2.6}$$

Substituting for u and v in (2.4) gives a single equation in one unknown, ψ.

SVERDRUP BALANCE

First consider the ocean interior.

Estimate magnitude of relative vorticity and planetary vorticity:

$$\frac{\left|\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right|}{f} = \frac{U}{fL} = \mathrm{Ro}$$

where Ro is the Rossby number.

Typical values: $U \sim 10^{-2}\mathrm{m\,s^{-1}}$, $L \sim 10^{6}\mathrm{m}$, $f \sim 10^{-4}\mathrm{s^{-1}}$

$\Rightarrow \mathrm{Ro} \sim 10^{-4} \ll 1.$

Thus $q \approx f = f_0 + \beta y$.

On the advective time-scale ($T \sim L/U$), the time-dependent term is also small, and friction is unlikely to important away from the boundaries.

$\Rightarrow$ in the ocean interior, (2.4) simplifies to:

$$\beta \frac{\partial \psi}{\partial x} = \frac{1}{\rho_0 H} \left\{ \frac{\partial \tau_s^{(y)}}{\partial x} - \frac{\partial \tau_s^{(x)}}{\partial y} \right\} \tag{2.7}$$

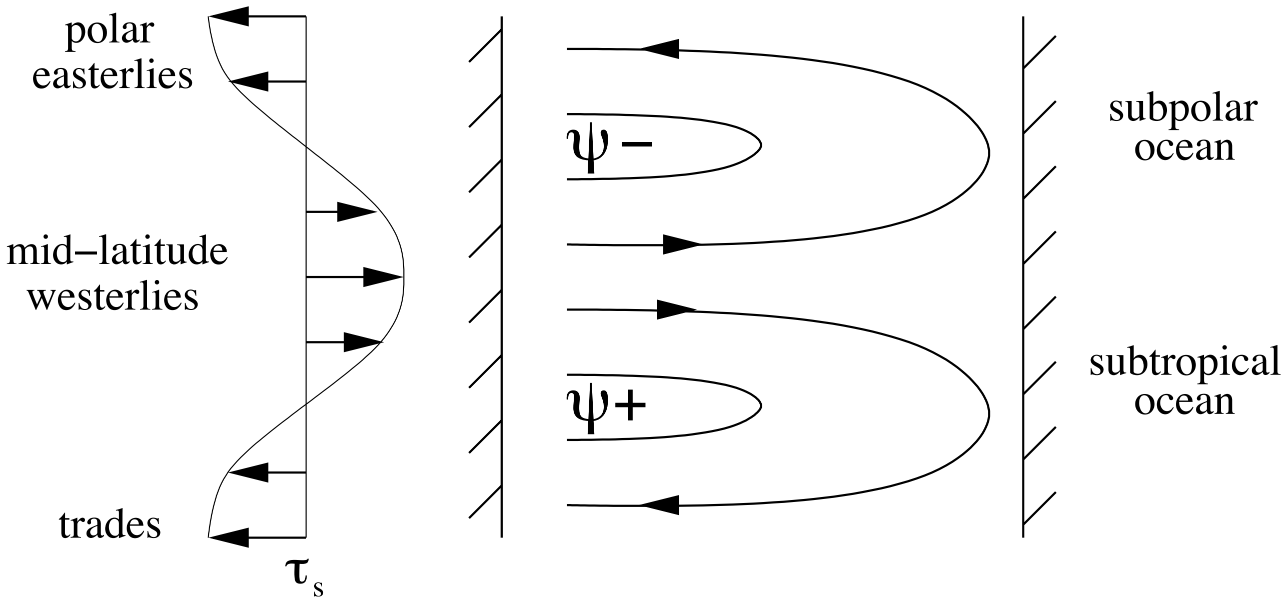
Sverdrup balance for a homogeneous ocean.

Local balance between advection of planetary vorticity and source of vorticity by wind-stress curl.

Boundary conditions?

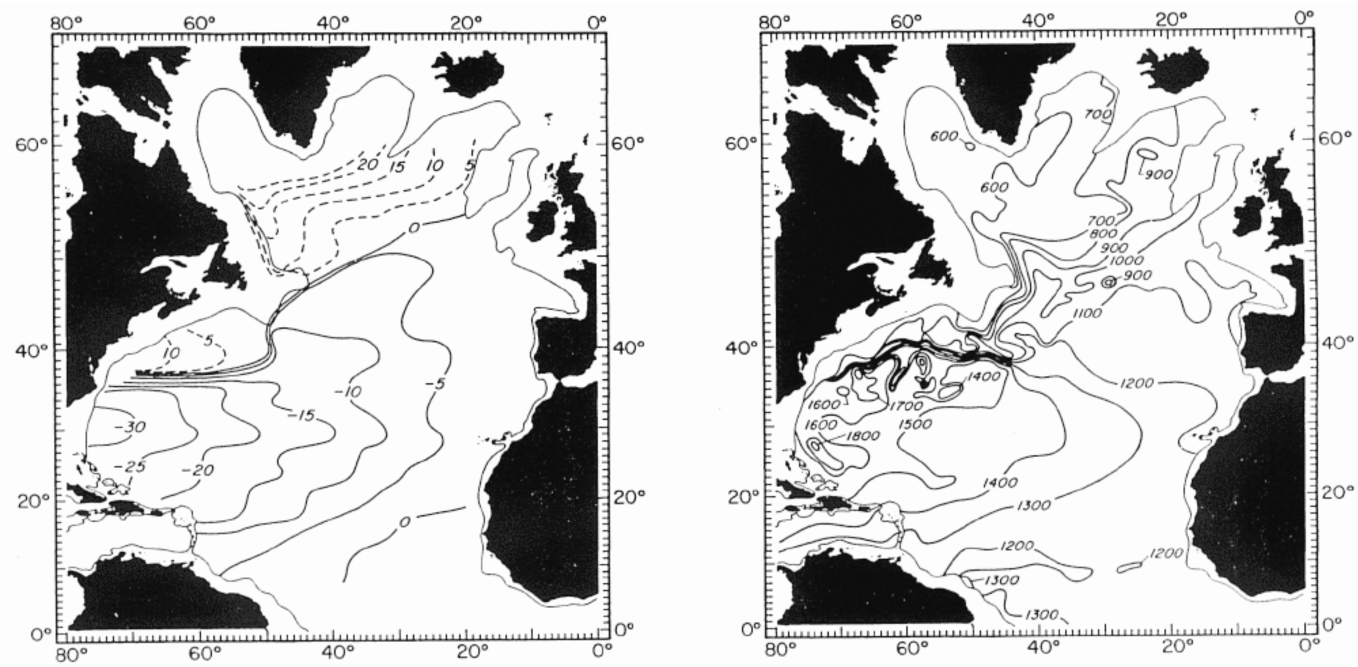
Would like to set $\psi = 0$ on *both* the western and eastern boundaries. However (2.7) is a 1st-order p.d.e. in x $\Rightarrow$ can satisfy only 1 b.c. in x.

Sverdrup noted that boundary currents tend to form on the western margins of ocean basins and applied the eastern boundary condition.

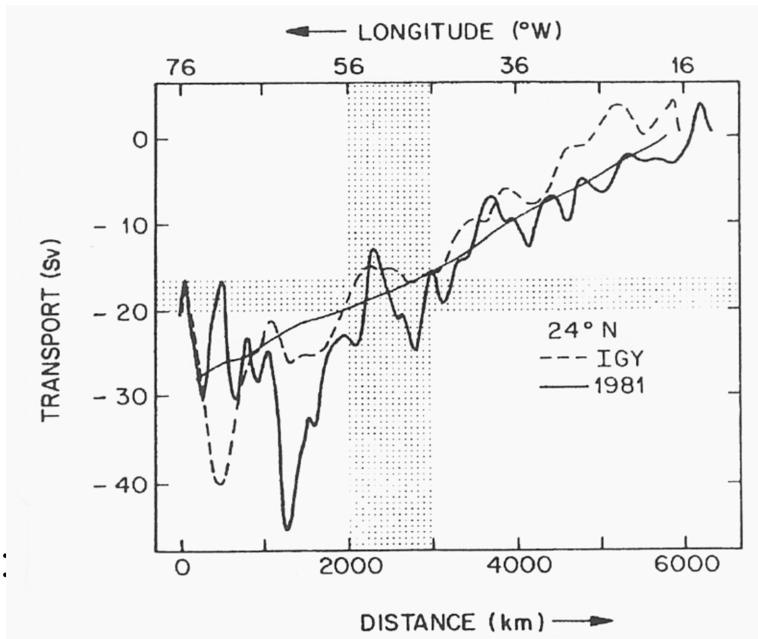


Predicts both subtropical and subpolar gyres.

Geostrophic transport from Sverdrup balance (Sv) and dynamic topography at 100m (mm) in the North Atlantic (from Gill 1982):



Comparison of Sverdrup transport across 24N in the Atlantic (thin solid line) and estimates of the transport from two hydrographic sections (from Schmitz et al. 1992):



WESTERN INTENSIFICATION

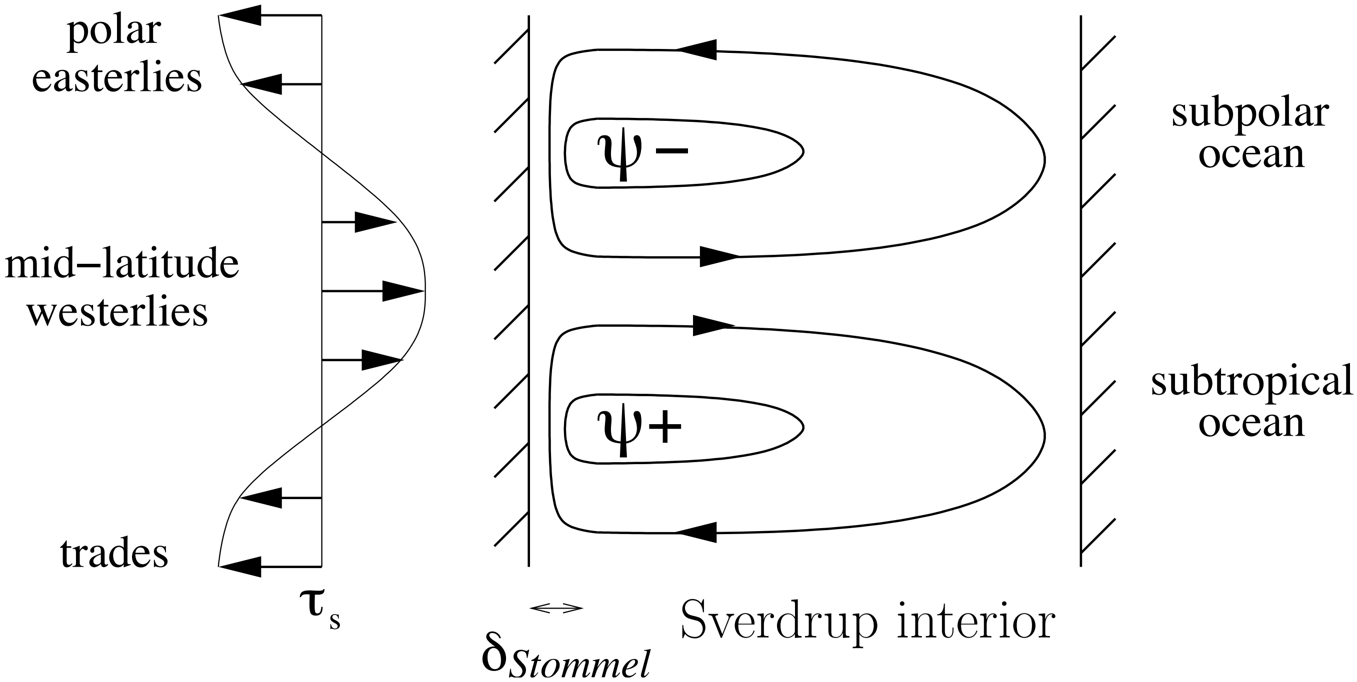
To close the circulation at western boundary requires additional physics.

Following Stommel (1948), introduce linear friction $\Rightarrow$

$$\beta \frac{\partial \psi}{\partial x} = \frac{1}{\rho_0 H} \left\{ \frac{\partial \tau_s^{(y)}}{\partial x} - \frac{\partial \tau_s^{(x)}}{\partial y} \right\} - r \nabla^2 \psi. \qquad (2.8)$$

Now a 2nd-order p.d.e. in x, allowing both western and eastern boundary conditions.

Solution in a rectangular basin with uniform zonal winds:



Boundary current: $$\beta \frac{\partial \psi}{\partial x} \sim -r \frac{\partial^2 \psi}{\partial x^2}$$

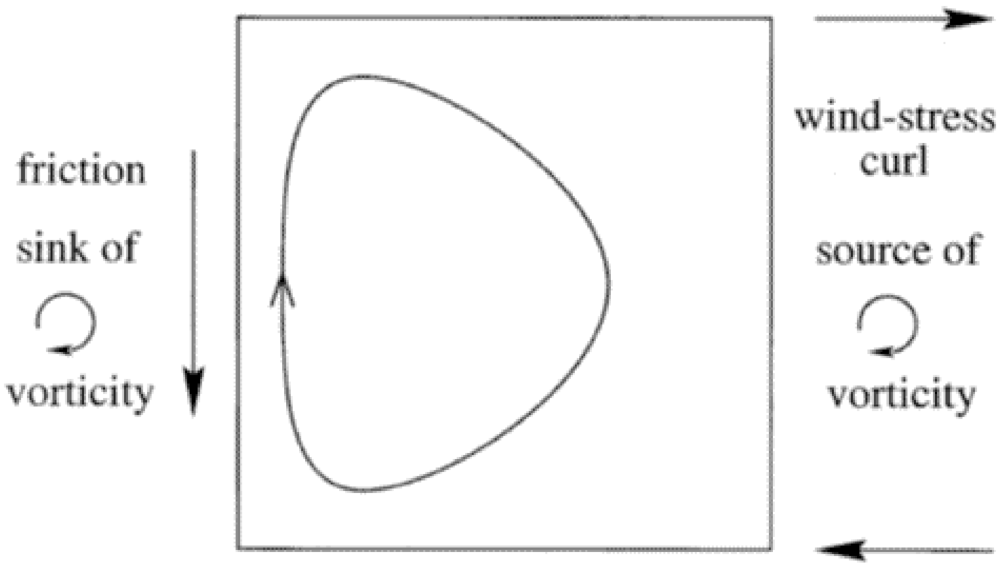
$\Rightarrow$ width $$\delta_{Stommel} \sim r/\beta$$

Why do the boundary currents form on the western margin?

Subtropical gyre: wind stress inputs anticyclonic vorticity
$\Rightarrow$ clockwise circulation in the Northern Hemisphere.

But the interior Sverdrup flow is equatorward
$\Rightarrow$ only consistent solution with western boundary current.

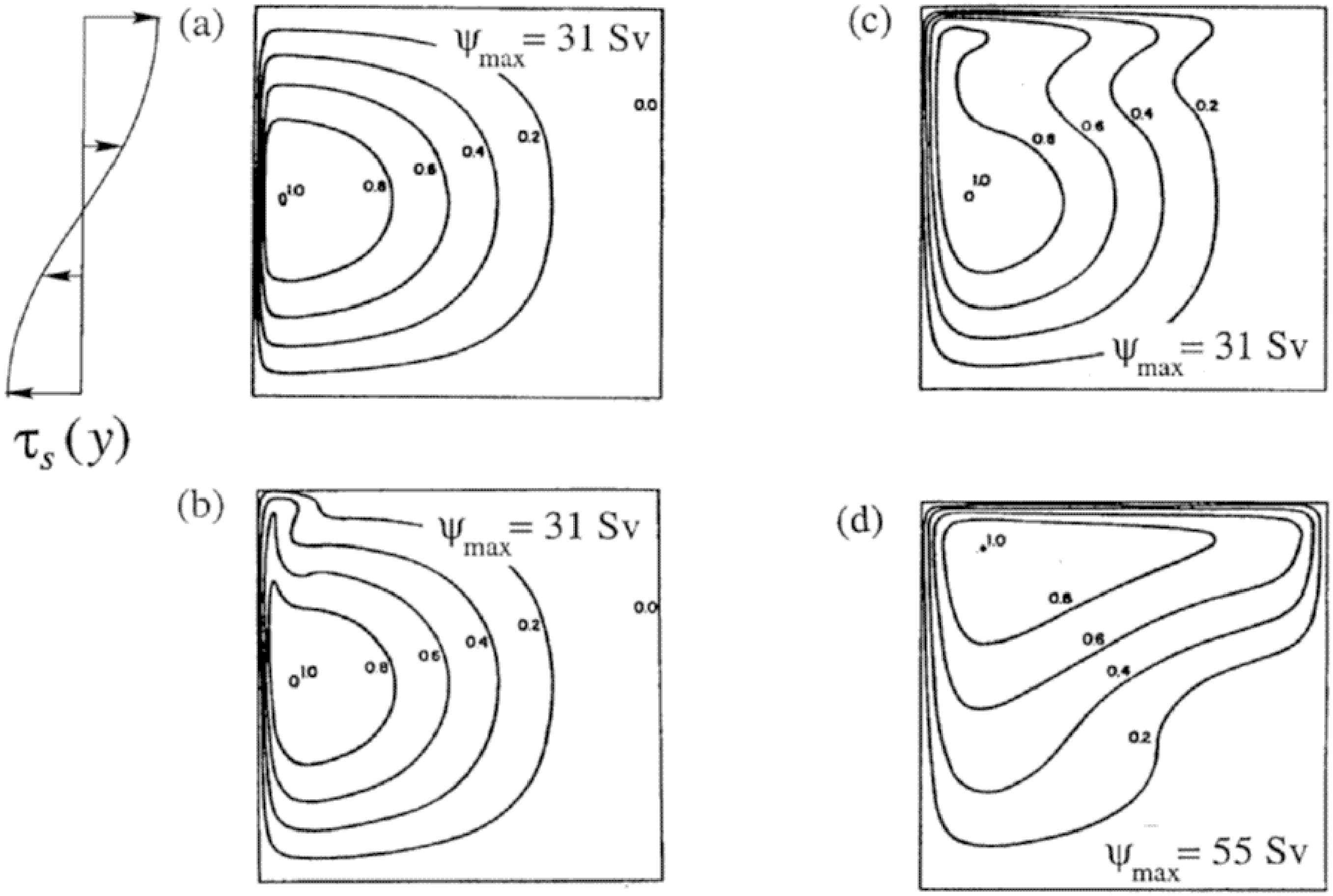
In a steady state, the source and sink of anticyclonic vorticity must balance:



NONLINEAR EFFECTS

In practice, relative vorticity is *not* negligible within the western boundary current.

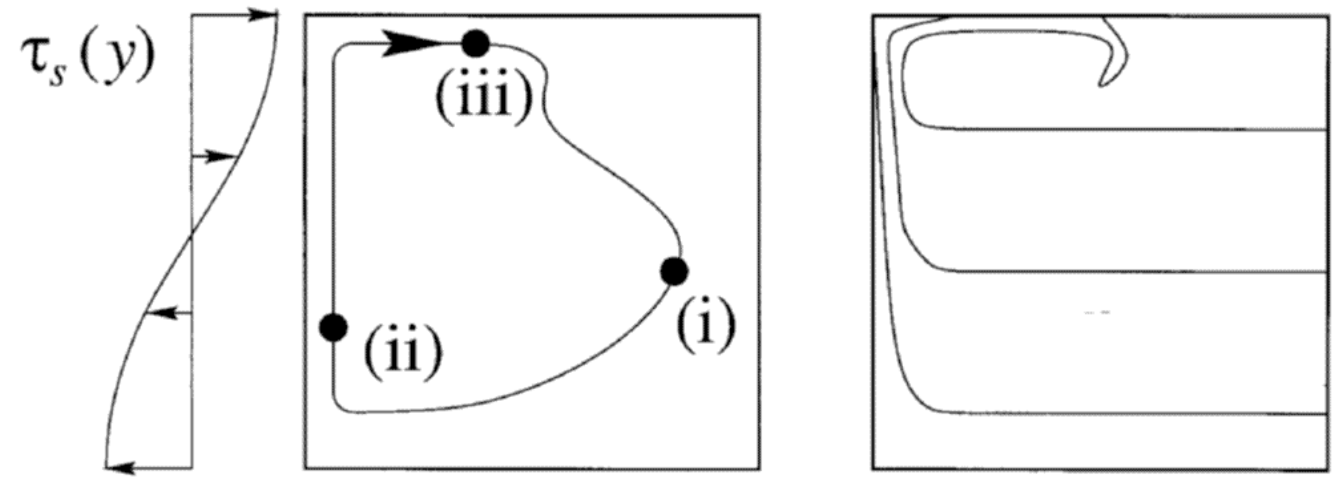
To include both friction and relative vorticity, it is necessary to resort to numerical solutions (adapted from Veronis 1966; also see Bryan 1963 for equivalent with lateral friction):



weak inertia $\rightarrow$ *strong inertia*

- boundary current extends along northern edge of gyre;
- gyre transport increases — known as *inertial recirculation.*

To obtain a physical understanding, consider sources and sinks of vorticity acting on a parcel of fluid as it travels around a closed streamline:



(i) vorticity source:

$$\mathbf{u}.\nabla q \approx \frac{1}{\rho_0 H}\mathrm{curl}\tau_s$$

(ii) vorticity redistribution:

$$\mathbf{u}.\nabla q \approx 0$$

(iii) vorticity sink:

$$\mathbf{u}.\nabla q \approx -r\nabla^2\psi$$

Nonlinear generalisation of Stommel gyre, but same underlying principle:

over a closed gyre circuit, net sources and sinks of vorticity must balance.

Can formalise by integrating vorticity equation [(2.4), with $\partial/\partial t = 0$] over area enclosed by a streamline, to give:

$$\frac{1}{\rho_0 H}\oint_{\psi} \tau_s.d\mathbf{l} - r\oint_{\psi} \mathbf{u}.d\mathbf{l} = 0 \tag{2.9}$$

(Niiler 1966).

What happens as $r \to 0$?

The only way (2.9) can be satisfied is if $\oint \mathbf{u}.d\mathbf{l}$ increases.

Either:

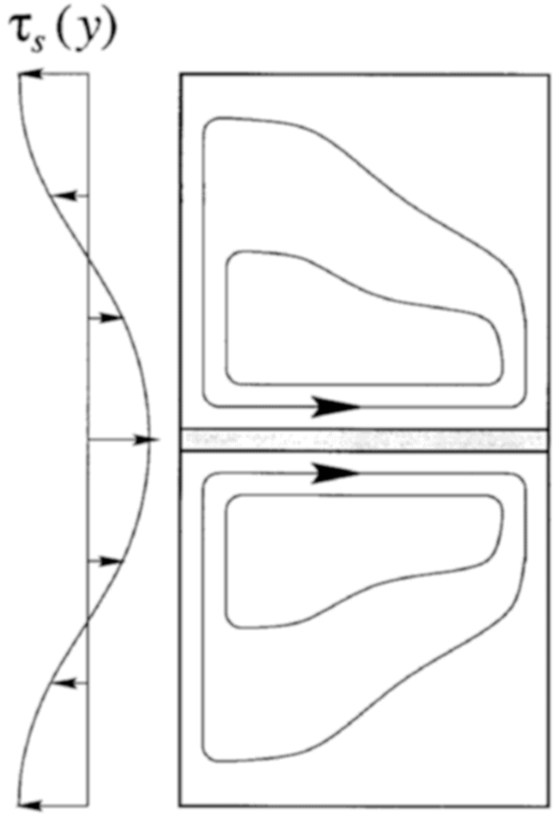
- the boundary current increases its length,
- or the velocities increase $\Rightarrow$ inertial recirculation;

cf. riding a bicycle with flat tyres (large friction) and fully inflated tyres (weak friction).

ROLE OF TRANSIENT EDDIES

So far we have considered only one (subtropical) gyre. Now consider a more "complete" model in which we have both a subtropical and subpolar gyre.

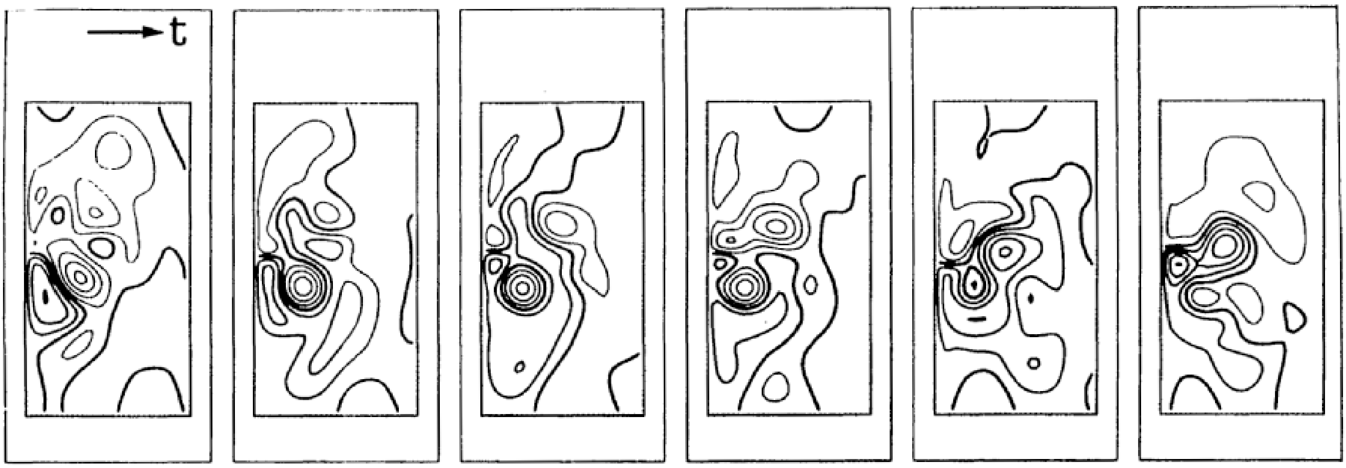
Initially, let's place an imaginary wall between the 2 gyres:



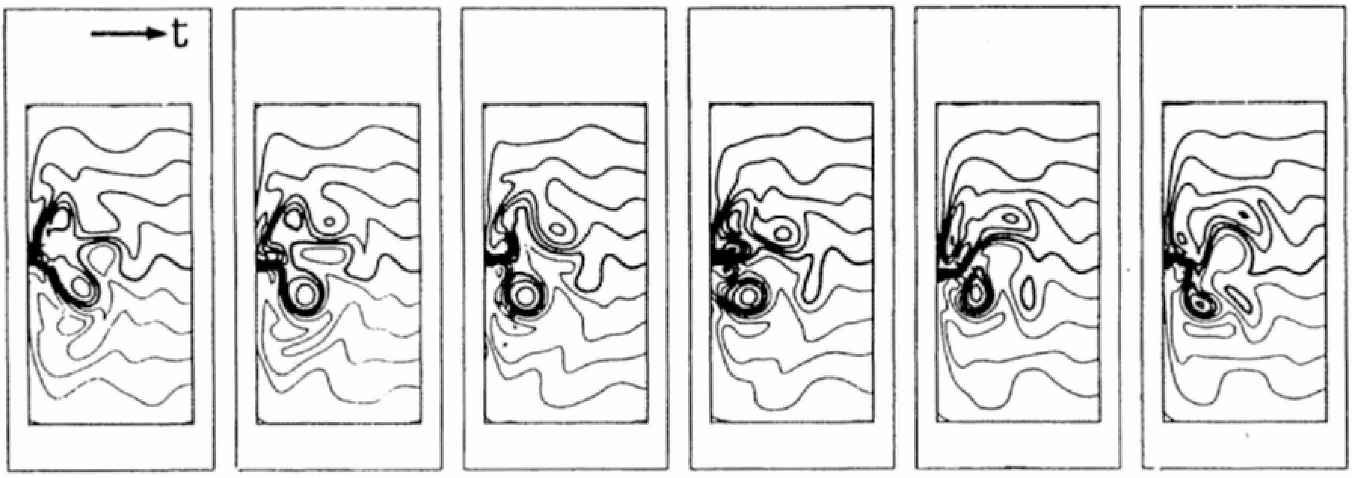
What happens if we remove the wall?

Numerical calculation (J. Marshall 1984):

streamfunction



vorticity



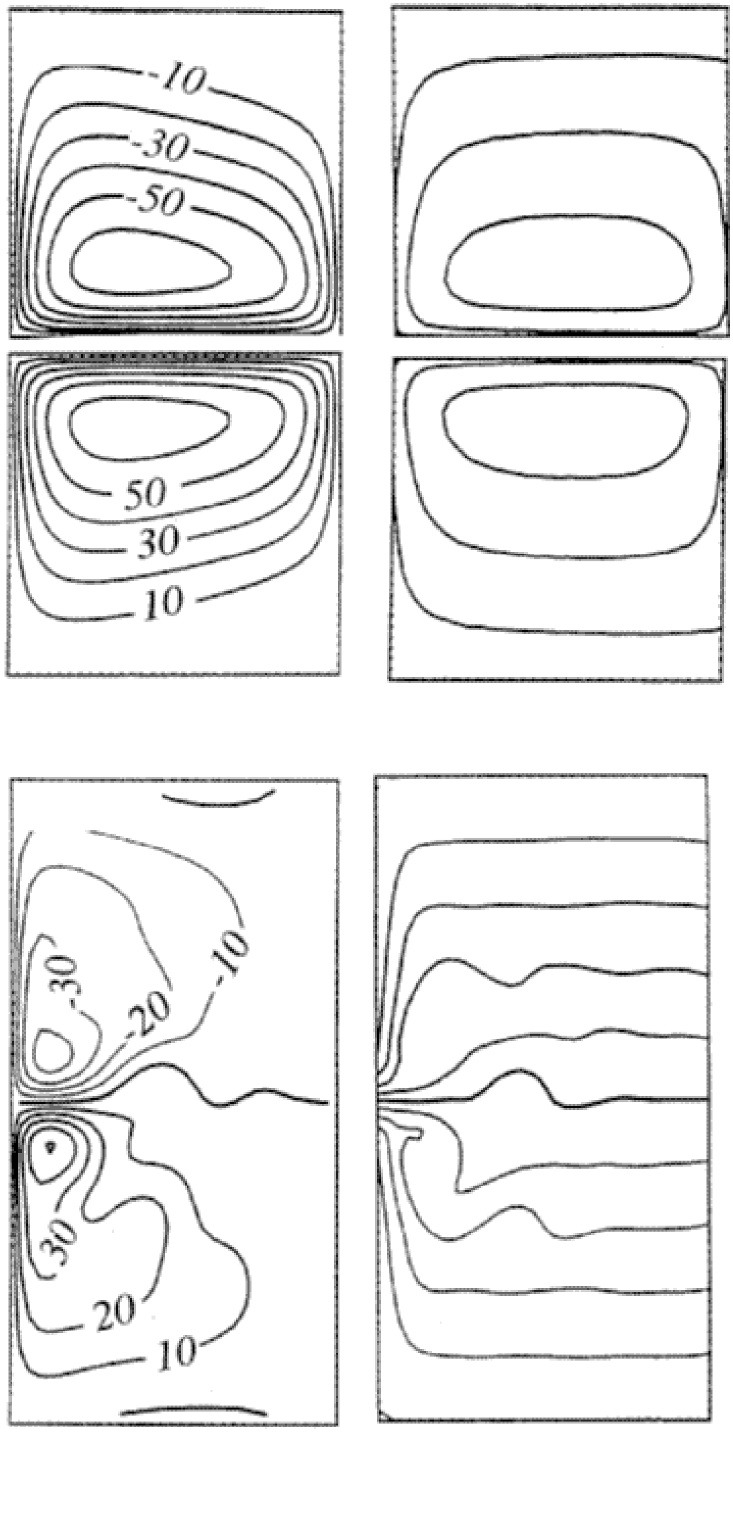
cf. Gulf Stream rings (figure, page 2-1)

Time-mean fields:

streamfunction *vorticity*

separate gyres

double gyre



Can can split the variables into mean and transient components:

$$\mathbf{u} \;=\; \overline{\mathbf{u}} + \mathbf{u}',$$

$$q \;=\; \overline{q} + q', ...$$

The time-mean vorticity equation is then

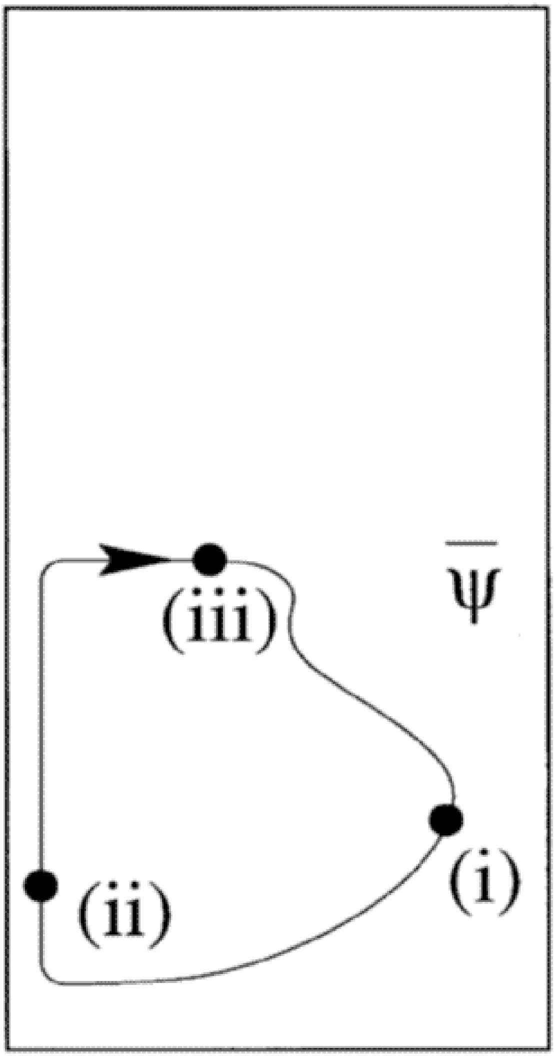
$$\overline{\mathbf{u}}.\nabla\overline{q} = \frac{1}{\rho_0 H}\left\{\frac{\partial\overline{\tau}_s^{(y)}}{\partial x} - \frac{\partial\overline{\tau}_s^{(x)}}{\partial y}\right\} - r\left(\frac{\partial\overline{v}}{\partial x} - \frac{\partial\overline{u}}{\partial y}\right) - \nabla.\overline{\mathbf{u}'q'}. \tag{2.10}$$

Finally, integrating this over the area enclosed by a time-mean streamline, we obtain:

$$\frac{1}{\rho_0 H}\oint_{\overline{\psi}} \overline{\tau}_s.d\mathbf{l} - r\oint_{\overline{\psi}} \overline{\mathbf{u}}.d\mathbf{l} - \oint_{\overline{\psi}} \overline{\mathbf{u}'q'}.d\mathbf{n} = 0. \tag{2.11}$$

Additional "sink" of vorticity in the time-mean vorticity equation associated with eddy vorticity fluxes.

Vorticity budget along a time-mean streamline:



(i) vorticity source:

$$\overline{\mathbf{u}}.\nabla\overline{q} \approx \frac{1}{\rho_0 H}\mathrm{curl}\overline{\tau}_s$$

(ii) vorticity redistribution:

$$\overline{\mathbf{u}}.\nabla\overline{q} \approx 0$$

(iii) vorticity "sink":

$$\overline{\mathbf{u}}.\nabla\overline{q} \approx -\nabla.\overline{\mathbf{u}'q'}$$

SUMMARY OF MAIN POINTS

- Have developed a simple homogeneous model of wind-driven gyres, with no vertical structure.
- Model is able to reproduce many features of the observed circulation, including:

  — subtropical and subpolar gyres,

  — western boundary currents,

  — inertial recirculation,

  — separated jets than meander and form rings.
- One reason for the success of the homogeneous model is that it captures the three essential ingredients of any ocean gyre:

  — a vorticity source,

  — a vorticity redistribution,

  — a vorticity sink.

  In the next lecture, we will see that these ideas carry over to a stratified ocean if one reinterprets q as the potential vorticity.

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General reading

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